Complex $\neq$ complicated

Squaring any 'normal' number = +ve

Central Idea:

New special number defined such that $j^2 = -1$

$\therefore \quad j = \sqrt{-1}$

Some use 'i' rather than 'j'

The existence of complex numbers is irrelevant: they're useful

↳ negative values also don't 'exist'

$x^2 - 1 = 0$

roots = ± 1

$x^2 + 1 = 0$

no 'real' solutions

but $\pm j$ are solutions

All numbers needed take form $\quad z = x + jy$

where

x, y are 'normal' real numbers

$j = \sqrt{-1}$

$x = \mathrm{Re}(z)$: x is real part of complex number
$y = \mathrm{Im}(z)$: y is imaginary part of complex number

$$(x_1 + jy_1) + (x_2 + jy_2) = (x_1 + x_2) + j(y_1 + y_2)$$

(add real numbers and add complex)

Opposite for subtraction

Complex Conjugate (or c.c) is defined by

$$\bar{z} = x - jy$$

Note:

$\mathrm{Re}(\bar{z}) = \mathrm{Re}(z)$

$\mathrm{Im}(\bar{z}) = -\mathrm{Im}(z)$

$z + \bar{z}$ is real, $z - \bar{z}$ is imaginary

jy and $\overline{jy}$ cancel (imaginary parts)

Note the imaginary part of z doesn't include the j:

e.g if $z = 3 - j2$

$\mathrm{Re}(z) = 3$ and $\mathrm{Im}(z) = -2$

If $x = 0 \longrightarrow$ purely imaginary
If $y = 0 \longrightarrow$ purely real

Multiplying Complex Numbers:

$(2+j)(3-2j)$
$= 6 + 3j - 2j^2 - 4j$

$\boxed{j^2 = -1}$

collect 'j' terms but $j^2 = -1$ so can be real value

$\therefore = 6 + 3j + 2 - 4j$
$= 8 - j$

Dividing Complex Numbers: $\left(\frac{z_1}{z_2}\right)$

Multiply by $\bar{z}_2$: $\dfrac{z_1 \bar{z}_2}{z_2 \bar{z}_2}$

e.g $\dfrac{2+j}{3-2j}$

$$\frac{(2+j)(3+2j)}{(3-2j)(3+2j)} = \frac{6 + 7j + 2j^2}{9 - 4j^2}$$

$$= \frac{4 + 7j}{13}$$

$\sqrt{-x} = \sqrt{-1}\sqrt{x}$

Where x is a positive number, the square roots of negative numbers can be represented as a number multiplied by $\sqrt{-1}$.

Thus $\sqrt{-121} = 11\sqrt{-1}$

and $\sqrt{-64} = 8\sqrt{-1}$

Complex numbers simplified the process of obtaining roots, e.g quadratic equation:

$$ax^2 + bx + c = 0$$

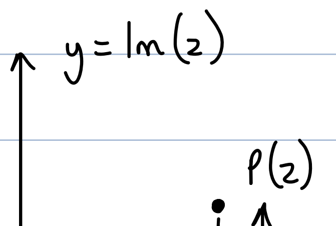
always has 2 roots

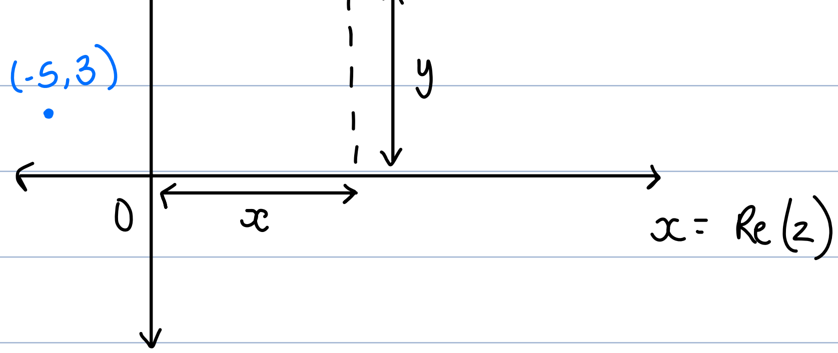
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

These roots are real numbers when $b^2 \geq 4ac$ and complex numbers when $b^2 < 4ac$.

The Argand Diagram:

Geometrically, complex numbers can be represented as points on a plane.
The number $z = x + jy$ represented by point P with coords (x, y), such diagram is Argand:
x- axis is called real axis
y- axis is called imaginary axis.





eg. $z = -5 + j3$

The Arithmetic of Complex Numbers:

Equality:

If two complex numbers are equal : ($z_1 = x_1 + jy_1$, is equal to $z_2 = x_2 + jy_2$) they are represented by the same points on the Argand diagram and

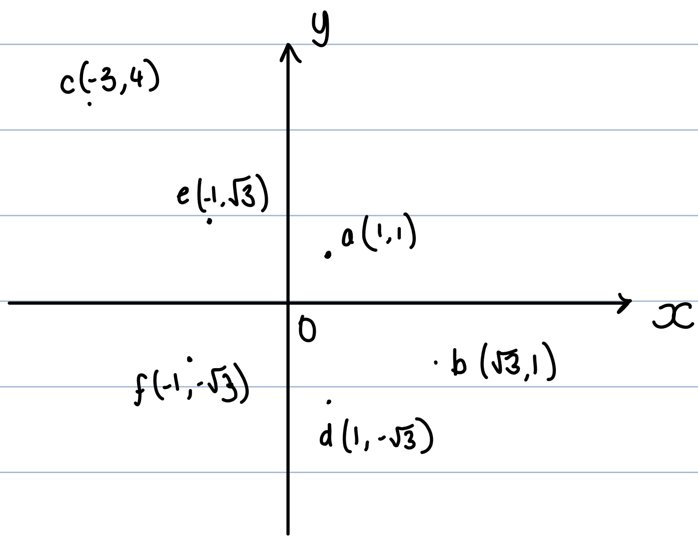
$$x_1 = x_2 \quad , \quad y_1 = y_2$$

Therefore, when two complex numbers are equal, we can equate their respective real and imaginary parts.

Examples:

3.2.5

1.



2. $z_1 = 1 + j2 \quad , \quad z_2 = 3 - j$

$z_1 + z_2 = 4 + j$

$z_1 - z_2 = j3 - 2$

$2z_1 - 3z_2 = 2 + j4 - 9 + 3j$

$= j7 - 7$

$5z_1 - 2z_2 = 5 + j10 - 6 + j2$

$= j12 - 1$

$2z_1 + z_2 = 5 + j3$

4.

$(6 - j3)(2 + j4) = 12 - j6 + j24 - 12j^2$

$j^2 = -1$

$= 24 + j18$

$(7 + j)(2 - j3) = 14 + j2 - j21 - 3j^2$

$= 17 - j19$

$(-1 + j)(-2 + j3) = 2 - j3 - j2 + 3j^2$

$= -1 - j5$

$(-3 + j2)(4 + j7) = -12 + j8 - j21 + j^2 14$

$= -26 - j13$

5.

$$\frac{4 - j6}{1 + j} = \frac{(4 - j6)(1 - j)}{(1 + j)(1 - j)} = \frac{4 - j6 - j4 + 6j^2}{1 - j^2}$$

$$= \frac{-2 - j10}{2}$$

$$= -1 - j5$$ ✓

b) $$\frac{(5+j3)}{(3-j2)} = \frac{(5+j3)(3+j2)}{(3-j2)(3+j2)} = \frac{15 + j10 + j9 + 6j^2}{9 + \cancel{6j} - \cancel{6j} - 4j^2}$$

$$= \frac{15 + j19 - 6}{9 + 4}$$

$$= \frac{9 + j19}{13}$$

$$= 9/13 + j\,19/13.$$ ✓

c) $$\frac{1-j}{4+j3} = \frac{(1-j)(4-j3)}{(4+j3)(4-j3)} = \frac{4 - j7 - 3}{16 + 9}$$

$$= \frac{1 - j7}{25}$$ ✓

d) $$\frac{-4 - j3}{2-j} = \frac{(-4-j3)(2+j)}{(2-j)(2+j)} = \frac{-8 - j10 + 3}{4 + 1}$$

$$= \frac{-5 - j10}{5}$$

$$= -1 - j2$$ ✓

6.

a) $(5+j3)(2-j) - (3+j)$

$= 10 - j5 + j6 - 3j^2 - 3 - j$

$= 10$

b) $(1-j2)^2$

$= 1 - j4 - 4 = -3 - j4$

c) $\dfrac{5-j8}{3-j4} = \dfrac{(5-j8)(3+j4)}{(3-j4)(3+j4)}$

$= 15 - j4 + 32$

$= \dfrac{47 - j4}{25}$

d) $\dfrac{1-j}{1+j} = \dfrac{(1-j)(1 \cdot j)}{(1+j)(1-j)} = -j$

e) $\frac{1}{2}(1+j)^2 = \frac{1}{2}(2j) = j$

f) $(3-j2)^2 = 9 - j12 - 4$

$= 5 - j12$

g) $\dfrac{1}{5-j3} - \dfrac{1}{5+j3} = \dfrac{5+j3}{(5-j3)(5+j3)} - \dfrac{5-j3}{(5+j3)(5-j3)}$

$= \dfrac{j6}{(5-j3)(5+j3)}$

$= \dfrac{j6}{25 + 9} = j\dfrac{3}{17}$

h) $\frac{1}{2} - \dfrac{3-j4}{5-j8}$

$$= \frac{1}{2} - \frac{(3-j4)(5+j8)}{(5-j8)(5+j8)} = \frac{1}{2} - \frac{15 + j4 + 32}{25 + 64}$$

$$= \frac{1}{2} - \frac{47 + j4}{89}$$

3.2.5 :

3.7) $f(x) = x^2 - 6x + 13$

$$x = \frac{6 \pm \sqrt{36 - 52}}{2} = \frac{6 \pm \sqrt{-16}}{2}$$

$$= 3 + 2j$$